# Sets

# Sets

Collection of elements.

- ▶ Empty set: $\varnothing$
- ▶ Integers: $\mathbb{Z}$
- ▶ Natural numbers: $\mathbb{N}$
- ▶ Real numbers: $\mathbb{R}$
- ▶ Rational numbers: $\mathbb{Q}$

# Cartesian Product

ordered paris of elements from multiple sets

# Subsets

If $A \subseteq B$, then no element of $A$ is not in $B$.

# Power Sets

Powerset of $A$, $\mathcal{P}(A)$ is the set of all subsets of $A$. There are $2^{|A|}$ subsets of $A$.

# Set Union

Like OR. $A \cup B$ is a set of all elements of either $A$ or $B$.

# Set Intersection

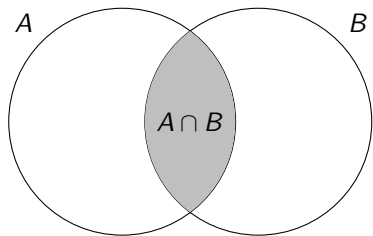Like AND. $A \cap B$ is the set of all elements that are in in both $A$ and $B$.

# Set Difference

$A - B$ is the set of all things in $A$ that are not in $B$.

# Complement

Universal set, aka, universe of discourse, $U$.
Complement of $A$, $\overline{A}$, is the set $U - A$.

# Venn Diagrams

# Proofs

▶ Proofs are chains of statements that lead from axioms to a final
statement, called a conclusion, such that if each statement in the
proof is true, the conclusion must be true.

## Truth
Mathematical logic does not deal with the meaning of truth. It only deals
with the systematic treatment of true and false statements. The meaning
of truth is a matter for philosophy.

# Theorems

- ▶ Theorems are statements that have been proved.
    - ▶ We usually use *theorem* to refer to major results in mathematics.
- ▶ Lemmas are theorems that are useful primarily in proving other theorems.
- ▶ Corrollaries are theorems that follow "quickly" (one or two deduction steps) from a theorem.

# Example: Context-free Grammars

Consider the following system of axioms:

- ▶ $S \rightarrow aSb$
- ▶ $S \rightarrow \epsilon$, where $\epsilon$ means empty string

The set of axioms above is in instance of a *grammar*. A grammar is

- ▶ a set of *terminal symbols*, which can be an *alphabet* (a.k.a. *lexicon*) or set of words; here $\{a, b\}$,
- ▶ a set of *nonterminal symbols* which act as syntactic vairables in the grammar; here $\{S\}$,
- ▶ a distinguished nonterminal that acts as the start symbol; here $S$, and
- ▶ a set of axioms of the form $\alpha \rightarrow \beta$, called *production* or *grammar* rules.

A *context-free grammar* contains production rules with only nonterminals on the left.

# Example: Language Generation

A production rule says that the symbol on the left can be replaced by the sequence of symbols on the right. A sequence of subsitutions consitutes a *proof* that the final sequence of symbols, the conclusion, is a sentence in the language specified by the grammar. For example

| | |
|---|---|
| *S* | the start symbol |
| *aSb* | by Rule 1, $S \rightarrow aSb$ |
| *aaSbb* | by Rule 1, $S \rightarrow aSb$ |
| *aabb* | by Rule 2, $S \rightarrow \epsilon$ |

is a proof that the sequence of symbols, or *string*, *aabb* is a sentence of the language, generated by successive applications of axioms, or production rules, in the langauge's grammar.

## Parsing

The opposite problem, determining whether a given string is a valid sentence in a language, can be solved by finding a sequence of production rules that produce the string. This sequence of production rules can be represented as a tree, called a *parse tree*, and the program that does this is called a *parser*. A parser is an essential component of language analyzers such as interpreters and compilers.

# Truth Functionals

An assignment of true or false to a new proposition is truth functional if the assignment is made based on the truth of related propositions, and the validity of the logical connectives linking the component propositions to the new proposition. In other words, the truth of a proposition is a *function* of the truth values of logically related propositions.