Counting

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The number of objects of interest.
At the end of this lesson you will

- understand the basic collections into which we can group objects, and
- know how to count several kinds of subsets of objects within collecions.

Lists

A list is an ordered sequence of elements denoted in literal form with objects – the elements or entries - enclosed in parentheses and separated by commas, e.g.:

$$(a, b, c)$$

Unlike sets, repetition is allowed and order matters:

$$\begin{aligned} (a, b, a) &\neq (a, b, b, a) \\ (a, b, c) &\neq (c, b, a) \end{aligned}$$

Remember that in sets "repeated" elements represent a single element and the order in which the elements are enumerated is not significant:

$$\begin{aligned} \{a, b, a\} &= \{a, b, b, a\} \\ \{a, b, c\} &= \{c, b, a\} \end{aligned}$$

Aside: Lists and Tuples in Programming

Programming languages typically distinguish between lists, which are mutable (elements can be changed), and tuples, which are immutable. The mathematical notion of a list we discuss here corresponds to the programming notion of a tuple, and the notation often mirrors the mathematical notation with the same semantics.

```
>>> ['a', 'b', 'c'] # A Python list
>>> ('a', 'b', 'c') # A Python tuple
>>> {'a', 'b', 'c'} # A Python set
```

Multiplication Principle

The number of possible lists of length n with a_1 possible choices for the first element, a_2 choices for the second element, and so on is the product

$$\prod_{i=1}^{n} a_i$$

Example:

If the set of vowels is $V = \{a, e, i, o, u\}$, the set of consonants, C, is the set of the remaining 21 letters in the 26-letter English alphabet, and every 3-letter word must start and end with a consonant with a vowel in the middle, then $|C| = 21$, $|V| = 5$ and the number of 3-letter words is $21 \cdot 5 \cdot 21 = 2205$.

Addition Principle

If a finite set X can be decomposed as a union $X = X_1 \cup X_2 \cup \cdots \cup X_n$, where $Xi \cap Xj = \emptyset$ whenever $i \neq j$, then $|X| = |X_1| + |X_2| + \cdots + |X_n|$.

Subtraction Principle

If X is a subset of a finite set U, then $|overlineX| = |U| - |X|$.

Factorials

If n is non-negative, then "n factorial", denoted $n!$ is

$$n! = \begin{cases} 1 & \text{if } n \leq 1 \\ n * (n-1)! & \text{otherwise} \end{cases}$$

Put another way, $0! = 1$, $1! = 1$, and for $n > 1$, $n! = n \dot{(} n-1) \dot{(} n-2) \cdots 1$

Permutations

A permutation of a set X is a non-repetitive list of the elements of X. If $|X| = n$, then the number of permutations of X is $n!$.

k-Permutations

A k-permutation of a set X is a non-repetitive k-element list of elements from X. If $|X| = n$, then the number of k-permutations of X is

$$P(n, k) = \frac{n!}{(n-k)!}$$

If $k > n$, then $P(n, k) = 0$.

Combination

A k-combination is a k-element subset of a set. If a set has n elements, then the number of k-combinations is

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$\binom{n}{k}$ is sometimes written $C(n,k)$.