

# Adversarial Search Review

## Artificial Intelligence

1. What is a zero-sum game?

**Solution:** A game in which the "score" for one agent is the negative of the other agent's, i.e., they add to zero.

2. What is another, perhaps better, term for zero-sum? Why?

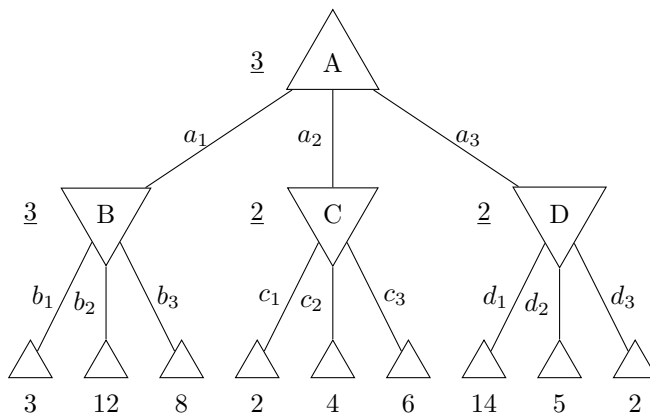
**Solution:** Equivalently, in a *constant sum* game the scores add to a constant value. For example, in chess each player gets a score of 1 or 0 in the case of a win, or each player gets  $\frac{1}{2}$  in the case of a draw. These scores always add to 1.

3. What is a ply?

**Solution:** One move by one player is called a ply. A ply for MAX plus the response ply for MIN constitute a game move.

4. In the following 2-ply minimax game tree, what are the minimax values of nodes A, B, C, and D, and which move is selected by MAX?

Move  $a_1$  is selected.



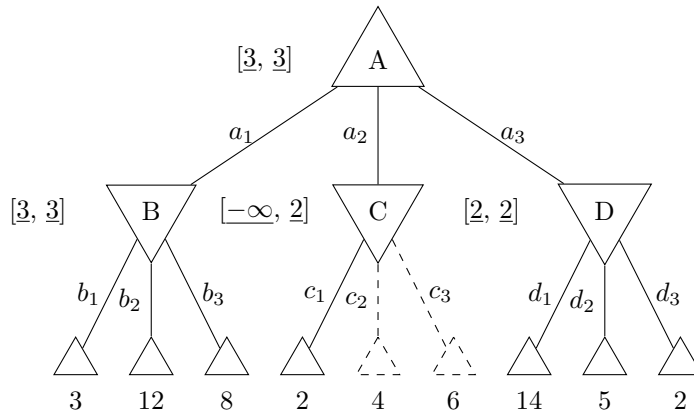
5. In the following game tree, what are the  $\alpha$  and  $\beta$  values in the intervals and which branches would be pruned from the tree with Alpha-Beta pruning?

Remember,

- $\alpha$  = the value of the best (i.e., highest-value) choice we have found so far at any choice point along the path for MAX. Think:  $\alpha$  = "at least."
- $\beta$  = the value of the best (i.e., lowest-value) choice we have found so far at any choice point along the path for MIN. Think:  $\beta$  = "at most."

At a MIN node, once we expand a child node that sets  $\beta$  to a value less than the highest  $\alpha$  value for a sibling node, we can stop expanding child nodes because they won't change the choice taken at the parent MAX node.

At a MAX node, once we expand a child node that sets  $\alpha$  to a value greater than the lowest  $\beta$  value for a sibling node, we can stop expanding child nodes because they won't change the choice taken at the parent MIN node.



6. In basic minimax search, the *Minimax* value function is defined by:

$$\text{Minimax}(s) = \begin{cases} \text{Utility}(s, \text{MAX}) & \text{if } \text{IsTerminal}(s) \\ \max_{a \in \text{Actions}(s)} \text{Minimax}(\text{Result}(s, a)) & \text{if } \text{ToMove}(s) = \text{MAX} \\ \min_{a \in \text{Actions}(s)} \text{Minimax}(\text{Result}(s, a)) & \text{if } \text{ToMove}(s) = \text{MIN} \end{cases}$$

How does the minimax algorithm and the *minimax* value function change when using a heuristic static evaluation function?

**Solution:** We add a depth parameter,  $d$ , which limits the number of plies of the game tree generated by the algorithm, and use this updated *Minimax* function:

$$\text{HMinimax}(s, d) = \begin{cases} \text{Eval}(s, \text{MAX}) & \text{if } \text{IsCutoff}(s, d) \\ \max_{a \in \text{Actions}(s)} \text{HMinimax}(\text{Result}(s, a), d + 1) & \text{if } \text{ToMove}(s) = \text{MAX} \\ \min_{a \in \text{Actions}(s)} \text{HMinimax}(\text{Result}(s, a), d + 1) & \text{if } \text{ToMove}(s) = \text{MIN} \end{cases}$$

7. In terminal states, what is the value of the  $\text{Eval}(s, \text{MAX})$  function?

**Solution:** In terminal states,  $\text{Eval}(\text{state}, \text{player}) = \text{Utility}(\text{state}, \text{player})$ , which is defined by the game rules.

8. How does Monte-Carlo Tree Search differ from Heuristic Alph-Beta Search?

**Solution:** Instead of searching to a given depth and applying a heuristic evaluation function to the resulting positions, we

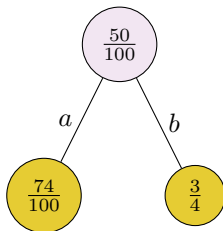
- simulate complete games (from a given position) to terminal positions, and
- back-up the win/loss scores up the tree.

9. Which weaknesses of Heuristic Alph-Beta Search does MCTS seek to overcome?

**Solution:** Two major weaknesses of heuristic alpha-beta tree search:

- Can't handle high branching factors. Go has a branching factor that starts at 361, which means alpha-beta search would be limited to only 4 or 5 ply.
- Can't always define a good static evaluation function. E.g., in Go material value is not a strong indicator and most positions are in flux until the endgame.

10. Using the *UCB1* upper confidence bound selection policy with a low *C* value, which path, *a* or *b*, would MCTS expand?



**Solution:** a